**MATHEMATICS METHODS**

**MAWA Semester 2 (Units 3 and 4)**

**Examination 2017**

**Calculator-Assumed**

# Marking Key

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The release date for this exam and marking scheme is

**the end of week 1 of term 4, 2017**

**Section Two: Calculator-assumed (99 Marks)**

**Question 10(a)**

|  |  |
| --- | --- |
| Solution  Isotope A decays faster.  Reason: Its half-life is less than the half-life of isotope B, i.e. it loses half of its mass faster than isotope B loses half of its mass. | |
| Marking key/mathematical behaviours | Marks |
| * answers correctly * uses the concept of half-life correctly | 1  1 |

**Question 10(b)**

|  |  |
| --- | --- |
| Solution  May assume that  and  where  and  are the amounts of isotopes A and B respectively, years from now.  Using the half-lives:  and .  So  and  When ,  (#)  i.e.  , i.e.  So it takes 1854 years before the ratio of the concentrations become 100 to 1. | |
| Marking key/mathematical behaviours | Marks |
| * uses exponential models for the amounts of isotopes at time * uses half-lives to solve for the constants  and  correctly * uses equation (#) * solves for the time, correct to the nearest year. | 1  1  1  1 |

**Question 11(a)**

|  |  |
| --- | --- |
| Solution  Population would be all the people eligible to vote in the election  Sample is the 100 voters asked | |
| Marking key/mathematical behaviours | Marks |
| * Identifies population correctly * Identifies sample correctly | 1  1 |

**Question 11(b)**

|  |  |
| --- | --- |
| Solution  Use a method to randomly choose 100 people from the electoral role | |
| Marking key/mathematical behaviours | Marks |
| * states a suitable method | 1 |

**Question 11(c)**

|  |  |
| --- | --- |
| Solution  For 100 estimate of proportion is 0.35  For 200  Std Dev =  = 0.03373    0.8618 | |
| Marking key/mathematical behaviours | Marks |
| * evaluates the standard deviation accurately * states distribution of correctly * Evaluates correct probability | 1  1  1 |

**Question 12(a)**

|  |  |
| --- | --- |
| Solution | |
| Marking Key/mathematical behaviours | Marks |
| * correctly drawn and labelled tree diagram * states the sample space | 1  1 |

**Question 12(b)**

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Solution   |  |  |  |  | | --- | --- | --- | --- | | **c** | 0 | 1 | 2 | | **Pr(C = c)** | 0.16 | 0.48 | 0.36 | | |
| Marking key/mathematical behaviours | Marks |
| * calculates correct probabilities (if only two correct, allow 1 mark) | 2 |

**Question 12(c)**

|  |  |
| --- | --- |
| Solution  *n* = 5  *p* = 0.84,  = 5(0.84)  = 4.2  The Bernesse family may expect to stop at least once, five times over the five days. | |
| Marking key/mathematical behaviours | Marks |
| * recognises the binomial distribution and correctly calculates the expected value | 1+1 |

**Question 13(a)**

|  |  |
| --- | --- |
| Solution  Pr (train is late 4 times out of 15) | |
| Marking key/mathematical behaviours | Marks |
| * recognises the binomial distribution and correctly calculates the expected value | 1+1 |

**Question 13(b)**

|  |  |
| --- | --- |
| Solution  Pr (train is late 4 times for at least 2 of the next 8 days): | |
| Marking key/mathematical behaviours | Marks |
| * calculation of probability of train not being late (using result from (a) * calculates probability for train late once * subtracts the two probabilities from one to achieve end result | 1  1  1 |

**Question 13(c)**

|  |  |
| --- | --- |
| Solution | |
| Marking key/mathematical behaviours | Marks |
| * recognizes ordered probability and uses appropriate calculation | 1 |

**Question 14(a)**

|  |  |
| --- | --- |
| Solution  Since  , where  is the noise level in decibels and  is the power and  is a reference power level, and since  increases by 10 if the power increases by a factor of 10, , (#)  So if  increases by a factor of 40,  increases by | |
| Marking key/mathematical behaviours | Marks |
| * obtains equation (#) or equivalent * obtains correct answer | 1  1 |

**Question 14(b)(i)**

|  |  |
| --- | --- |
| Solution  Since  it follows that  (#)  i.e. | |
| Marking key/mathematical behaviours | Marks |
| * obtains approximation (#) * obtains correct answer | 1  1 |

**Question 14(b)(ii)**

|  |  |
| --- | --- |
| Solution  Since  and  it follows that  (#)  and so | |
| Marking key/mathematical behaviours | Marks |
| * evaluates  correctly * obtains approximation (#) * obtains correct answer | 1  1  1 |

**Question 15(a)**

|  |  |
| --- | --- |
| Solution  and  (#)  and so  and  Since the period is 1 year, i.e. 365 days, | |
| Marking key/mathematical behaviours | Marks |
| * obtains equations (#) * solves for  and  correctly * obtains correct value for | 1  1  1 |

**Question 15(b)**

|  |  |
| --- | --- |
| Solution  When  we have  (#)  i.e.  i.e.  So the 356th day, (December 22nd) will be the longest day. | |
| Marking key/mathematical behaviours | Marks |
| * obtains equation (#) * obtains correct answer | 1  1 |

**Question 15(c)**

|  |  |
| --- | --- |
| Solution    So  when  (#)  i.e. when  i.e.  So the number of daylight hours will be decreasing fastest on the 82nd day, i.e. on March 23rd. | |
| Marking key/mathematical behaviours | Marks |
| * differentiates correctly * obtains equation (#) * obtains correct answer | 1  1  1 |

**Question 15(d)**

|  |  |
| --- | --- |
| Solution    By the increments formula  and so if  So the largest difference in the number of daylight hours in successive days is 0.043 hours, i.e. 2.6 minutes. | |
| Marking key/mathematical behaviours | Marks |
| * correctly calculates * uses increments formula correctly | 1  1 |

**Question 16(a)**

|  |  |
| --- | --- |
| Solution  (i)  (ii)  (iii) | |
| Marking key/mathematical behaviours | Marks |
| * recognises exact probabilities are equal to zero * calculates correct probability * applies the appropriate formula and associated probabilities leading to the correct answer and correct diagram | 1  1  1+1+1 |

**Question 16(b)**

|  |  |
| --- | --- |
| Solution | |
| Marking key/mathematical behaviours | Marks |
| * states probability condition involving * calculates the correct value for | 1  1 |

**Question 16(c)**

|  |  |
| --- | --- |
| Solution | |
| Marking key/mathematical behaviours | Marks |
| * uses the correct formula and substitutes values * calculation the standard score * states the correct answer | 1  1  1 |

**Question 17(a)**

|  |  |
| --- | --- |
| Solution  cos(2t) + c  *c* = 5.5 | |
| Marking key/mathematical behaviours | Marks |
| * correctly integrates to find equation for *v* involving *c* * correctly evaluates *c* * writes an expression for *v* | 1  1  1 |

**Question 17(b)**

|  |  |
| --- | --- |
| Solution    When or - 2  When 2  = 13.57 m or 9.57 m | |
| Marking key/mathematical behaviours | Marks |
| * determines correct integral of function plus c * calculates a value for c * calculates *x* accurately when *t* = 2 and includes both possible values | 1  1  1+1 |

**Question 18(a)**

|  |  |
| --- | --- |
| Solution | |
| Marking key/mathematical behaviours | Marks |
| * identifies the parameters of the standard normal and states the problem in terms of probability * states the correct result | 1  1 |

**Question 18(b)**

|  |  |
| --- | --- |
| Solution | |
| Marking key/mathematical behaviours | Marks |
| * applies the normal distribution to determine *x* * states the correct result for *k* | 1  1 |

**Question 18(c)**

|  |  |
| --- | --- |
| Solution  The *x*-value of 6 is 2.4 standard deviations away from the mean. | |
| Marking key/mathematical behaviours | Marks |
| * provided an acceptable explanation | 1 |

**Question 18(d)**

|  |  |
| --- | --- |
| Solution | |
| Marking key/mathematical behaviours | Marks |
| * evaluates the correct integral * defines *F(x)* * states the three domains correctly for *F(x)* | 1  1  1 |

**Question 19**

|  |  |
| --- | --- |
| Solution  Check sample size is large enough for normal approximation *np* >10 and  *n(1 – p)* > 10.  In this case, 1000 x 0.48 = 480 > 10  1000 x 0.52 = 520 > 10  Therefore, normal approximation can be applied.    =  = (0.45, 0.51)  (0.45, 0.51) is a 95% Confidence Interval for the true proportion of students excited by the upcoming concert. | |
| Marking key/mathematical behaviours | Marks |
| * Checks the sample size for normal approximation * Sets up CI and evaluates correctly * correctly interprets result | 1+1  1+1  1 |

**Question 20(a)**

|  |  |
| --- | --- |
| Solution        The area is approximately 20 square units.   1. The area represents the distance travelled by the projectile between | |
| Marking key/mathematical behaviours | Marks |
| * estimates the function at the values suggested (allow  ) * applies the summation correctly * states the required area * correctly interprets the meaning of the area as the distance travelled | 2  1  1  1 |

**Question 20(b)**

|  |  |
| --- | --- |
| Solution  The area of the triangle formed by and the (between )  = 1 square unit.  Hence,   1. region A = | |
| Marking key/mathematical behaviours | Marks |
| * Calculates the area of the triangle * Calculates the area of region A * Defines region B in terms of integrals of * Re-arranges the integrals using the integral properties so as to be able to use the information given * Shows the required result. | 1  1  1  2  1 |

**Question 21(a)**

|  |  |
| --- | --- |
| Solution | |
| Marking key/mathematical behaviours | Marks |
| * obtains  as the antiderivative * evaluates at limits correctly | 1  1 |

**Question 21(b)**

|  |  |
| --- | --- |
| Solution | |
| Marking key/mathematical behaviours | Marks |
| * obtains  as the antiderivative * evaluates at limits correctly | 1  1 |

**Question 21(c)(i)**

|  |  |
| --- | --- |
| Solution  Rate of heat loss is a maximum at    Maximum rate of heat loss is ~ 3 kilojoules per day. | |
| Marking key/mathematical behaviours | Marks |
| * states correct values of * states the maximum rate of heat loss | 1+1  1 |

**Question 21(c)(ii)**

|  |  |
| --- | --- |
| Solution:    The heat loss is ~293 kilojoules. | |
| Marking key/mathematical behaviours | Marks |
| * indicates that the heat loss in the integral from 0 to 120 of * states the correct result * states the correct units | 1  1  1 |

**Question 21(c)(iii)**

|  |  |
| --- | --- |
| Solution | |
| Marking key/mathematical behaviours | Marks |
| * indicates solving the integral of from 0 to 120 of  = 300 * states the correct result | 1  1 |

**Question 22(a)**

|  |  |
| --- | --- |
| Solution | |
| Marking key/mathematical behaviours | Marks |
| * Clearly shows the correct intercepts * Minimum and maximum points are reasonably accurate * Graph is appropriately smooth | 1  1  1 |

**Question 22(b)**

|  |  |
| --- | --- |
| Solution  Using the CAS calculator to solve for *a:*    From the graph in part (a) it is obvious that  so, need to select | |
| Marking key/mathematical behaviours | Marks |
| * Solves correctly (if provides additional values for *a* – subtract one mark) | 2 |

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*12 Cobbler Place, MIRRABOOKA 6061*